Integrálás

Határozatlan (Anti-derivált)

$f(x)$ fgv prim. fgv-e $F(x)$, ha $F'(x) = f(x)$ $\forall x \in$ vmi intervallum.
ha F prim fgv $\Rightarrow$ $F+C$ prim. fgv ($c \in \mathbb{R}$), ebben az esetben
$\int f = \{F + c;\ c \in \mathbb{R}\}$

a, c konstans, $f, g: \mathbb{R} \to \mathbb{R}$ fgv

- $\int a f(x) + c g(x)\,dx = a \cdot \int f(x)\,dx + c \cdot \int g(x)\,dx$
- $\mu \in \mathbb{R} - \{-1\}$ $\int x^{\mu}\,dx = \frac{x^{\mu+1}}{\mu+1} + c$
- $\int x^{-1}\,dx = \ln|x| + c$
- $\int \sin(x)\,dx = -\cos(x) + c$, $\int \cos(x)\,dx = \sin(x) + c$,
  $\int e^x\,dx = e^x + c$, $\int N^x\,dx = \frac{1}{\ln(N)} N^x + c$

$$\int f(x)^{\mu} \cdot f'(x)\,dx = \frac{f^{\mu+1}(x)}{\mu+1} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| - c$$

1. FELADAT

a) $\int 6x^7 + 9x^4\,dx =$

b) $\int \frac{3x^4 - 2x^2}{\sqrt[3]{x}}\,dx =$

c) $\int 7^{x-3}\,dx =$

d) $\int 7\sin(x) - e^x\,dx =$

2. FELADAT

Intro: $\int \frac{12x^2 - 4x + 6}{4x^3 - 2x^2 + 6x - 8}\,dx =$

a) $\int \frac{9x^2}{3x^3 - 7}\,dx =$

b) $\int \frac{x^2}{3x^3 - 7}\,dx =$

d) $\int \frac{1}{x \cdot \ln(x)}\,dx =$

e) $\int (x+7)^{12}\,dx =$

f) $\int \sin^9(x) \cdot \cos(x)\,dx =$

g) $\int x\sqrt{3 + 2x^2}\,dx =$

$$\int f(ax+b)\,dx = \frac{1}{a} F(ax+b) + c$$

3. FELADAT

a) $\int \sin\left(2x + \frac{\pi}{2}\right)\,dx =$

b) $\int 4^{3x+7}\,dx =$

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

4. FELADAT

a) $\int x\cdot\cos(x)\,dx =$

b) $\int (3x^2-2x+6)e^x\,dx =$

* c) $\int x^k \cdot \ln(x)\,dx =$

d) $\int \sin(x)\cdot\cos(x)\,dx =$

e) $\int e^x\cos(x)\,dx =$

f) $\int \operatorname{arctg}(x)\,dx =$

g) $\int \arcsin(x)\,dx =$

$$\left(\begin{aligned} \operatorname{arctg}'(x) &= \frac{1}{1+x^2} \\ \arcsin'(x) &= \frac{1}{\sqrt{1-x^2}} \end{aligned}\right)$$

h) $\int \cos^n(x) =$

i) $\int \sin^n(x) =$

$$\sin^2 x + \cos^2 x = 1$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$
$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$
$$\sin x \sin y = -\frac{1}{2}[\cos(x+y) - \cos(x-y)]$$

5. FELADAT tripon. fgv-ek linearizálással.

a) $\int \sin^2(x)\,dx =$

b) $\int \cos^3(x)\,dx =$

c) $\int \sin(x) \cdot \sin(2x)\,dx$

d) $\int \sin^3(x)\cos^2(x)\,dx =$

e) $\int \sin^5(x)\cos^2(x)\,dx =$

__Helyettesítés:__ $\int f(g(x)) \cdot g'(x)\,dx = \int f(x)\,dx$

A gyakorlatban:

__Pl.__: $\int \sqrt{1-x^2}\,dx \underset{\substack{x = \sin(t) \\ \frac{dx}{dt} = \cos(t)}}{=} \int \sqrt{1-\sin^2(t)}\cos(t)\,dt \underset{\substack{\text{olyan intervallumon,} \\ \text{ahol } \cos(t) > 0}}{=} \int \cos^2(t)\,dt = \frac{1}{2}\left(t + \sin(t)\cos(t)\right)$

$= \frac{1}{2}\arcsin(x) + x\underbrace{\cos(\arcsin(x))}_{\sqrt{1-x^2}}$

VEGYES FELADATOK

a) $\int \sin^3(7x)\cos(7x)\,dx$

b) $\int \frac{\sin^3(x)}{1-\cos(x)}\,dx$

c) $\int (2x-3)^{100}\,dx$

d) $\int \sin^3(x)\,dx$

e) $\int \frac{\ln^2(x)}{x}\,dx$

f) $\int \frac{1}{\sqrt{\sin(x)\cos^3(x)}}\,dx$

g) $\int \frac{1}{x^3\sqrt{\ln(x)}}\,dx$

h) $\int \frac{x}{\sqrt{1-x^2}}\,dx$

i) $\int \sqrt[3]{1-3x}\,dx$

j) $\int \frac{e^{2x}}{1+e^{2x}}\,dx$

k) $\int \frac{x}{x^2+a^2}\,dx$

l) $\int \frac{1}{\mathrm{sh}(x)}\,dx$

m) $\int \frac{dx}{2+3x^2}$

n) $\int \frac{1}{3x^2+6x+1}\,dx$

o) $\int e^x \sin^2(x)\,dx$

HELYETTESÍTÉS:

a) $\int \frac{1}{\sqrt{(1-x^2)^3}} =$

b) $\int \frac{1}{x\sqrt{x^2+a}} =$

H: a: $\sin(t)=x$ b: $t=a/x$